Topic: Limits at infinity (horizontal asymptotes)

Question: Find the limit.

$$\lim_{x\to\infty} \frac{6}{4x^2}$$

Answer choices:

A $\frac{3}{2}$

B 0

C ∞

D 1

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Solution: B

When we need to evaluate a limit at infinity, we look at how our function is behaving as we approach infinity.

For this problem, we can take our limit

$$\lim_{x\to\infty} \frac{6}{4x^2}$$

and see how it behaves as $x \to \infty$. As x increases, the denominator of our fraction will increase but our numerator will stay the same. When we have a fraction that has a constant in the numerator and a variable in the denominator that increases to infinity, we'll see that the fraction approaches zero. This means that

$$\lim_{x\to\infty} \frac{6}{4x^2} = \frac{6}{4(\infty)^2}$$

$$\lim_{x\to\infty} \frac{6}{4x^2} = \frac{6}{\infty}$$

$$\lim_{x\to\infty} \frac{6}{4x^2} = 0$$

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Topic: Limits at infinity (horizontal asymptotes)

Question: Find the limit.

$$\lim_{x\to\infty} \frac{6x^3+2x^2-x+1}{8x^3-1}$$

Answer choices:

A -1

B ∞

C $\frac{3}{4}$

D $\frac{3}{8}$

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Solution: C

If we try to substitute directly into our limit, we'll get ∞/∞, which is a non-real solution. Which means we'll need to manipulate the limit in order to solve it. The first thing we can do is to divide the numerator and the denominator by the variable with the highest power, in this case x^3.

$$\lim_{x\to\infty}\frac{6x^3+2x^2-x+1}{8x^3-1}=\lim_{x\to\infty}\frac{6x^3+2x^2-x+1}{8x^3-1}\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)$$

$$\lim_{x\to\infty}\frac{6x^3+2x^2-x+1}{8x^3-1}=\lim_{x\to\infty}\frac{\frac{6x^3}{x^3}+\frac{2x^2}{x^3}-\frac{x}{x^3}+\frac{1}{x^3}}{\frac{8x^3}{x^3}-\frac{1}{x^3}}$$

$$\lim_{x\to\infty}\frac{6x^3+2x^2-x+1}{8x^3-1}=\lim_{x\to\infty}\frac{6+\frac{2}{x}-\frac{1}{x^2}+\frac{1}{x^3}}{8-\frac{1}{x^3}}$$

Evaluate the limit.

$$\lim_{x\to\infty}\frac{6x^3+2x^2-x+1}{8x^3-1}=\frac{6+\frac{2}{\infty}-\frac{1}{\infty}+\frac{1}{\infty}}{8-\frac{1}{\infty}}$$

Remember as x increases, the denominator of a fraction will increase but the numerator will stay the same. When we have a fraction that has a constant in the numerator and an increasingly large value in the denominator, the fraction will approach zero.

$$\lim_{x\to\infty}\frac{6x^3+2x^2-x+1}{8x^3-1}=\frac{6+0-0+0}{8-0}$$

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$$\lim_{x\to\infty}\frac{6x^3+2x^2-x+1}{8x^3-1}=\frac{3}{4}$$

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Topic: Limits at infinity (horizontal asymptotes)

Question: Find the limit.

$$\lim_{x\to\infty} \frac{x^2+1}{\sqrt{5x^4+2x^2}}$$

Answer choices:

A $\sqrt{5}$

B 0

C ∞

D $\frac{\sqrt{5}}{5}$

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Solution: D

If we try to substitute directly into our limit, we'll get ∞/∞, which is a non-real answer. We'll need to manipulate the limit in order to solve it. The first thing we can do is bring our numerator into the square root of the denominator. We can do this by first squaring the numerator and then taking its square root.

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\lim_{x\to\infty}\frac{\sqrt{\left(x^2+1\right)^2}}{\sqrt{5x^4+2x^2}}$$

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\lim_{x\to\infty}\sqrt{\frac{\left(x^2+1\right)^2}{5x^4+2x^2}}$$

Now we can expand the numerator of the fraction.

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\lim_{x\to\infty}\sqrt{\frac{x^4+2x^2+1}{5x^4+2x^2}}$$

At this point, we can apply another property of limits and bring the limit into the square root.

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\sqrt{\lim_{x\to\infty}\frac{x^4+2x^2+1}{5x^4+2x^2}}$$

We can use L'Hospital's rule,

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

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to manipulate our limit further. Remember, we can use L'Hospital's rule as many times as we need to until evaluating the limit gives us a real-number answer.

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\sqrt{\lim_{x\to\infty}\frac{4x^3+4x}{20x^3+4x}}$$

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\sqrt{\lim_{x\to\infty}\left(\frac{4}{4}\right)\frac{x^3+x}{5x^3+x}}$$

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\sqrt{\lim_{x\to\infty}\frac{x^3+x}{5x^3+x}}$$

If we try to evaluate the limit at this point, we still get ∞/∞. Apply L'Hospital's rule again, we get

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\sqrt{\lim_{x\to\infty}\frac{3x^2+1}{15x^2+1}}$$

Since we still get ∞/∞, we'll apply L'Hospital's rule one more time and get

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\sqrt{\lim_{x\to\infty}\frac{6x}{30x}}$$

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\sqrt{\lim_{x\to\infty}\frac{1}{5}}$$

Evaluate the limit.

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$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\sqrt{\frac{1}{5}}$$

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\frac{1}{\sqrt{5}}$$

$$\lim_{x\to\infty}\frac{x^2+1}{\sqrt{5x^4+2x^2}}=\frac{\sqrt{5}}{5}$$

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